TWO-DIMENSIONAL MAGNETIC FIELDS IN MAGNETO-HYDRODYNAMIC CHANNELS WITH STEEL WALLS AT FINITE MAGNETIC REYNOLDS NUMBERS

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Plane problems on the distribution of a two-dimensional magnetic field in magnetohydrodynamic channels with ferromagnetic walls at appreciable magnetic Reynolds numbers and prescribed flow hydrodynamics are studied. An integral representation for the total magnetic induction is constructed with the use of a complex influence function describing the field resulting from a unit current. This makes it possible to obtain arbitrarily close approximations to exact solutions of the problems on a digital computer. Influence functions for various channels can be determined by mirror reflections and conformal mappings. The method is illustrated by numerical calculations of the distribution of the magnetic field for the flow of a conducting fluid along a plane ferromagnetic wall and the flow of a fluid in the space between ferromagnetic walls. Calculations are carried out on the effect of an external circuit and an inhomogeneous transverse velocity profile on the distribution of the magnetic field.

In conducting magnetohydrodynamic (MHD) channels with finite dimensions, the magnetic field is in general three-dimensional when the magnetic Reynolds number (R_m) is appreciable. However, if the external magnetic field $\mathbf{B}_e$ is parallel to the electrode walls and end effects are suppressed (for instance, by longitudinal, nonconducting partitions at the entrance and exit), then, subject to certain restrictions on the geometry of the current taps and the distributions of the velocity u and the conductivity σ, the equation for the electric potential when $R_m \gg 1$ has a solution [1] corresponding to a homogeneous electric field. Such a case is realized, for instance, when the entire surfaces of the parallel electrodes are covered with sufficiently long, highly conducting current taps, pointing in the direction of the z axis, normal to the planes of the electrodes, and u and σ are independent of z. In such a channel the distribution of the total two-component magnetic field is two-dimensional: $\mathbf{B} = (B_x, B_y, 0)$ (the current is in the z direction).

The induction $\mathbf{B}$ can be found by the method of representing B_x and B_y in series form and matching the solutions at the walls of the channel [2].

More general ways of solving plane MHD problems when the magnetic permeability is homogeneous ($\mu = \mu_0 =$ const) involve the formulation of an integral equation for the transverse component of $\mathbf{B}$, using the Green function for the magnetic vector potential [3,4], or the direct insertion of Ohm's law into the Biot–Savart law [5]. By the latter method it is possible to solve certain problems for plane-meridian fields in which the construction of the Green's function for the vector potential is difficult [6].

If the lateral, nonconducting walls of the channel are bordered with a steel conductor of magnetism, the two-dimensional formulation of the problem on the distribution of $\mathbf{B}$ becomes more rigorous. Therefore the restrictions on the geometry of the electrodes in plane problems can be relaxed and the influence of the external circuit on the distribution of $\mathbf{B}$ in the channel will be different in its dependence on the orientation of the current tapes. However, when steel walls are present the integral representation of $\mathbf{B}$ becomes more complicated. For some MHD channels of canonical form the effect of steel walls can be taken into

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account by the method of reflection of the currents in the channel and use of the Biot–Savart law, counting the reflected currents as real also. Application of conformal mapping makes it possible to widen the class of problems that admit of an integral representation for **B**.

1. Flow of a Conducting Fluid along a Plane Ferromagnetic Wall. Let the fluid flow along a wall with magnetic permeability μ_2, situated in the region $q \le 0$ of the complex plane $s = p + iq$. Everywhere above the wall $\mu = \mu_1$. The quantities μ_2 and μ_1 can be commensurate (when the wall is made of saturated or magnetically stable steel). For $q > 0$ there is a plane external magnetic field $B_e = B_{ep} + iB_{eq}$. Currents flow in the region $0 \le p \le l$,* $0 \le q \le \delta(p)$, where $\delta(p)$ is the upper wall of the channel or the free boundary of the stream, and close symmetrically with respect to the working zone as $|s| \to \infty$. The conductivity $\sigma(s)$ and the velocity $u(s) = u_p + iu_q$ of the fluid are prescribed. It is required to find the total magnetic field $B(s)$ for $q > 0$. Such a problem may be encountered in the flow of a conducting fluid in an open flume with a ferromagnetic floor [7], in the study of pellicular MHD flows [8], pumps with unidirectional inductors, etc.

The boundary conditions require the continuity of B_q and B_p/μ on $q = 0$, the continuity of $-\infty < p < \infty$, and B_p and B_q on the remaining boundaries of the region containing currents, and attenuation of B_i as $|s| \to \infty$. Account can be taken of the influence of the steel wall if for every current $di = j(s')dp'dq'$ in $q > 0$ a fictitious reflected current di_0 in $q < 0$ is constructed so that the boundary conditions are satisfied. The magnetic self-field $dB_i(s)$ at a point $s(q > 0)$ of a current $j(s')dp'dq'$ at the point $s'(q > 0)$ is

$$dB_i = L(s, s')\, j(s')\, dp'dq', \quad L(s, s') = L_p + iL_q \tag{1.1}$$

where $L(s, s')$ is a certain influence function that takes account of the real and the reflected current. Using Sirl's well-known solution of the problem (see, for instance [9]), we obtain

$$L_p = \frac{1}{2\pi}\left[\frac{q'-q}{(q-q')^2+(p-p')^2} - \frac{\mu_2-\mu_1}{\mu_2+\mu_1}\,\frac{q+q'}{(q+q')^2+(p-p')^2}\right] \tag{1.2}$$

$$L_q = \frac{1}{2\pi}\left[\frac{p-p'}{(q-q')^2+(p-p')^2} + \frac{\mu_2-\mu_1}{\mu_2+\mu_1}\,\frac{p-p'}{(q+q')^2+(p-p')^2}\right] \tag{1.3}$$

The field $B_i(s)$ of all the currents in the fluid is determined as

$$B_i(s) = B(s) - B_e(s) = \int_s L(s, s')\, j(s')\, dp'dq' \tag{1.4}$$

Inserting Ohm's law $j = \sigma[E + \mathrm{Im}(\bar{u}B)]$ into this, we arrive at the complex integral equation

$$B(s) - \int_s L(s, s')\sigma(s')\,\mathrm{Im}\,[\overline{u(s')}\,B(s')]\, dp'dq' = B_e(s) + \int_s L(s, s')\sigma(s')\, E\, dp'dq' \tag{1.5}$$

If E is independent of s, then, under the conditions that $\sigma(s)$, $u(s)$, and E are bounded and that $L(s, s')$ is integrable, Eq. (1.5) is of Fredholm's type. It is equivalent to a system of two real integral equations for B_p and B_q, which can be solved by numerical methods. If (1.2) and (1.3) are inserted into Eq. (1.5), the latter is equivalent to the Biot–Savart law in which the real currents in the fluid [first terms in (1.2) and (1.3)] are taken into account as well as the reflected currents [second terms in (1.2) and (1.3)]. It is characteristic that as $s' \to s$ the second terms in (1.2) and (1.3) vanish and the function $L(s, s')$ becomes the same as that for a line current in a nonmagnetic medium:

$$L(s, s') = \frac{1}{2\pi}\exp\left(i \operatorname{arc\,ctg}\frac{p-p'}{q-q'}\right)[(q-q')^2+(p-p')^2]^{-1/2}$$

From this it follows that the singularity of the kernel of the integral equation (1.5) is a simple pole and consequently the Fredholm theory is valid for (1.5). If $u_p \gg u_q$, (1.5) reduces to a single real Fredholm integral equation with an improved kernel singularity

$$B_q(s)\left[1 - R_m \int_0^1 \int_0^{\delta(p)} L_q(s, s')\sigma(s')u_p(s')\, dp'dq'\right] \tag{1.6}$$

*The currents are normal to the s plane.

$$- R_m \int_0^1 \int_0^{\delta(p)} L_q(s, s') \sigma(s') u_p(s') [B_q(s') - B_q(s)] \, dp' dq'$$
$$= B_{eq}(s) + R_m E \int_0^1 \int_0^{\delta(p)} L_q(s, s') \sigma(s') \, dp' dq'$$

and a definite integral for $B_p(s)$

$$B_p(s) = R_m \int_0^1 \int_0^{\delta(p)} [L_p(s, s') \sigma(s') u_p(s') B_q(s') + E L_p(s, s') \sigma(s')] \, dp' dq' \tag{1.7}$$

Here all linear dimensions are divided by l, B by B_0, σ by σ_0, u by u_0, E by $u_0 B_0$, where reference quantities are distinguished by the subscript zero, and $R_m = \mu_0 \sigma_0 u_0 l$.

Using relations (1.6) and (1.7), we can obtain arbitrarily close approximations to exact solutions of the problem by numerical methods.

2. Flow in a Strip. The fluid moves in an infinite strip $0 \le y \le \delta$ (δ = const) with the velocity $u_x(x,y)$ in the field $B = B_x + iB_y$. We have $\mu = \mu_0$ in $0 < y < \delta$, $\mu = \infty$ in $y \ge \delta$, $y \le 0$. Currents flow in the region $0 \le x \le l$, $0 \le y \le \delta$ and close through an external circuit, arranged either symmetrically to, or to the left or right of the working zone.

This model corresponds, for instance, to the quasisteady motion of a short, longitudinal cluster between long, linear electrodes placed in the gap of an electromagnet with steel poles, or to the motion of an axisymmetric annular piston in a small gap between coaxial steel walls. If continuous flow in a channel is considered, then the presence of nonconducting longitudinal partitions in the regions $x < 0$, $x > l$ is assumed. In the last case the extension of the steel walls outside the region $0 \le x \le l$ will be limited. However, as indicated by approximate estimates, the effect of the edge of a pole is practically absent in a gap of width δ at distances larger than δ from the edge when $R_m \le 10$ [5].

We assume at first that the external circuit connected to the electrodes is arranged symmetrically with respect to x about the working region. Then the boundary conditions for the problem are the following: $B_x = 0$ on $y = 0$, $y = \delta$; B_x and B_y are continuous on the boundaries $x = 0$, $x = l$; and $B_{iy}|_{x=\infty} = -B_{iy}|_{x=-\infty}$. As in Sec. 1, the problem can be reduced to an integral equation if an influence function in a relationship of the type (1.1) can be found. Construction of $L(z, z')$ can be carried out by the method of reflections [9]; however, a more direct way involves the conformal mapping of the gap $0 \le y \le \delta$ in the plane $z = x + iy$ on the upper half-plane $s = p + iq$ by means of the function $s = \exp(\pi z/\delta)$. This enables one to make immediate use of solutions obtained earlier. As the influence functions L are analytic everywhere in the z and s planes with the exception of the points $s = s'$, $z = z'$, they can be expressed in terms of a complex potential Φ

$$L_s(s, s') = \overline{\frac{d\Phi(s)}{ds}}, \quad L_z(z, z') = \overline{\frac{d\Phi(z)}{dz}} \tag{2.1}$$

(the bar denotes complex conjugate).

Because of the invariance of the Laplace operator we have $\Phi(z) = \Phi[s(z)]$ whence, after differentiation and conversion to conjugate values, we obtain

$$L_z(z, z') = L_s(s, s') \overline{\frac{ds}{dz}}, \quad L_z = L_x + iL_y$$
$$L_x = -\frac{1}{4\delta}\left[\frac{\sin\pi(y+y')/\delta}{\operatorname{ch}\pi(x-x')/\delta - \cos\pi(y+y')/\delta} + \frac{\sin\pi(y-y')/\delta}{\operatorname{ch}\pi(x-x')/\delta - \cos\pi(y-y')/\delta}\right] \tag{2.2}$$

$$L_y = \frac{1}{4\delta}\left[\frac{\operatorname{sh}\pi(x-x')/\delta}{\operatorname{ch}\pi(x-x')/\delta - \cos\pi(y+y')/\delta} + \frac{\operatorname{sh}\pi(x-x')/\delta}{\operatorname{ch}\pi(x-x')/\delta - \cos\pi(y-y')/\delta}\right] \tag{2.3}$$

It is evident that the form of the function $L(z, z')$ ensures that all boundary conditions of the problem are satisfied.

Asymptotic estimates enable one to isolate singular terms in (2.2) and (2.3), these having singularities of the same order as those that occur in problems that do not involve steel walls.

Suppose now that the external circuit is not symmetric with respect to the channel, and is located at a sufficient distance from it. In the problem under consideration the influence of the external circuit cannot be neglected since B_i does not die out as $x \to \pm \infty$. From physical considerations it follows that the remote external circuit affects only the distribution of B_y in the channel, but not that of B_x [this, in particular, is evident from (2.2) and (2.3) if $x' \to \pm \infty$].

Every current element in the working zone, flowing into the external circuit (which also comprises the electrodes and the current taps), produces a certain homogeneous, transverse field in the working zone, of which account can be taken by writing

$$L_y^{(e)} = L_y + \gamma/4\delta$$

where L_y is defined by (2.3) and $\gamma = \text{const}$. If the external circuit is closed to the right of the channel the entire current to the right of $x = -\infty$ vanishes and consequently $L_y^{(e)}\big|_{x=-\infty} = 0$ when $\gamma = 2$. If the external circuit is closed to the left of the channel, then $\gamma = -2$.

When constructing an integral equation for B_y on the basis of relationships of the type (1.4) and (1.5), one must keep in mind the fact that the electric field strength E depends on the operating conditions of the channel and on the parameters of the external circuit and cannot be prescribed arbitrarily. Let the external circuit have a total conductivity g_e and a source of emf ε, opposed to the emf of the channel. Then, according to the method of nodal potentials of circuit theory, we have

$$E = -\frac{\varkappa}{hg_i}\int_0^1\int_0^\delta u_x B_y \sigma dx dy - \frac{\varepsilon}{h}(1-\varkappa), \quad g_i = \frac{1}{h}\int_0^1\int_0^\delta \sigma dx\, dy \tag{2.4}$$

where g_i is the internal conductivity of the channel, h is the distance between the electrodes, and $\varkappa = g_i/(g_i + g_e)$. We shall convert to dimensionless parameters by dividing ε by $u_0 B_0 h$, and g_i by $\sigma_0 l^2/h$ (see Sec. 1). In accordance with (1.5) and (2.4) the distribution of B_y in the channel is described by the following Fredholm equation with kernel having a simple pole and improved singularity:

$$B_y(z)\Big[1 - R_m\int_0^1\int_0^\delta K(z, z')dx'dy'\Big] - R_m\int_0^1\int_0^\delta K(z, z')[B_y(z') - B_y(z)]\, dx'dy'$$
$$= B_{ey}(z) - R_m\varepsilon(1-\varkappa)\int_0^1\int_0^\delta \sigma(z') L_y^{(e)}(z, z')\, dx'dy' \tag{2.5}$$

where the kernel is

$$K(z, z') = \sigma(z') u_x(z')\Big[L_y^{(e)} - \frac{\varkappa}{g_i}\int_0^1\int_0^\delta \sigma(z') L_y^{(e)} dx'dy'\Big]$$

For $\varepsilon \gg 1$ the channel consumes power from the external circuit and operates in the propellent (pumping) mode. When $0 \le \varepsilon \ll 1$ a drain of power from the channel into the external circuit occurs.

If $\sigma = \sigma(x)$ and $u_x = u_x(x)$ the problem has a one-dimensional solution for B_y since $B_x = 0$ everywhere. In fact, in this case the well-known equation for the inductions gives for B_x

$$\nabla^2 B_x - \mu\sigma u \frac{\partial B_x}{\partial x} = 0 \tag{2.6}$$

As $B_x = 0$ on the walls and at $x = \pm\infty$, it follows from the uniqueness of the solution of the Dirichlet problem for (2.6) that $B_x \equiv 0$.

3. Flow in an Exit Cone. The fluid moves in a plane, conical diffusor with ideally ferromagnetic lateral walls* at $\varphi \le 0, \varphi \ge \varphi_0 = \pi/n$ ($n = \text{const}$), $0 \le \rho < \infty$. Production of a conducting flow in the channel is accomplished either by the introduction of the working medium through small holes in the walls or by a chemical reaction in the neighborhood of $\rho = 0$. The velocity of the medium is directed radially: $u_\rho(\rho, \varphi)$; electric currents flow in the region $\rho_0 \le \rho \le \rho_0 + l$ in a direction perpendicular to the plane of the problem (axisymmetric model) or through an external circuit as $\rho \to \infty$.

*Polar coordinates are employed.

The boundary conditions are similar to those in the previous problem with the difference that now the self-field B_i of the currents in the channel dies out at infinity. Mapping the channel sector in the plane of $\omega = \xi + i\eta = \rho e^{i\varphi}$ on the strip $0 \le y \le \delta$ in the z plane by the function $z = (\delta/\pi) n \ln \omega$ and using the expressions (2.2) and (2.3), we obtain

$$L_\rho(\omega, \omega') = -\frac{n}{4\pi\rho}\left\{\frac{\sin n(\varphi+\varphi')}{0.5\,[(\rho/\rho')^n + (\rho'/\rho)^n] - \cos n(\varphi+\varphi')} + \frac{\sin n(\varphi-\varphi')}{0.5\,[(\rho/\rho')^n + (\rho'/\rho)^n] - \cos n(\varphi-\varphi')}\right\}$$

$$L_\varphi(\omega, \omega') = \frac{n}{4\pi\rho}\left\{\frac{(\rho/\rho')^n - (\rho'/\rho)^n}{(\rho/\rho')^n + (\rho'/\rho)^n - 2\cos n(\varphi+\varphi')} + \frac{(\rho/\rho')^n - (\rho'/\rho)^n}{(\rho/\rho')^n + (\rho'/\rho)^n - 2\cos n(\varphi-\varphi')}\right\}$$

As the velocity of the medium is radial, the problem reduces to an integral equation for B_φ and a definite integral for B_ρ.

4. Flow in a Half-Strip. The problem corresponds to the model of Sec. 2, but the magnetic circuit is now closed on the left of the working zone: $\mu = \mu_0$ in the half-strip $x > 0$, $0 < y < \delta$ and $\mu = \infty$ outside the half-strip. Currents flow in the region $x_0 < x < x_0 + l$, $0 < y < \delta$ and are closed at infinity. The boundary conditions correspond to those of the problem of Section 2, but for every current $di' = j(z')dx'dy'$ in the fluid, its magnetic field $dB_{iy}(x)$ at large values of x is equal to $\mu_0 di'/\delta$ and $dB_{ix} \to 0$. This permits us to construct $L(z, z') = L_x + iL_y$ in the form

$$L_x = -\frac{1}{4\delta}\left[\frac{\sin\pi(y+y')/\delta}{\operatorname{ch}\pi(x+x')/\delta - \cos\pi(y+y')/\delta} + \frac{\sin\pi(y-y')/\delta}{\operatorname{ch}\pi(x+x')/\delta - \cos\pi(y-y')/\delta} + \frac{\sin\pi(y+y')/\delta}{\operatorname{ch}\pi(x-x')/\delta - \cos\pi(y+y')/\delta} + \frac{\sin\pi(y-y')/\delta}{\operatorname{ch}\pi(x-x')/\delta - \cos\pi(y-y')/\delta}\right]$$

$$L_y = \frac{1}{4\delta}\left[\frac{\operatorname{sh}\pi(x+x')/\delta}{\operatorname{ch}\pi(x+x')/\delta - \cos\pi(y+y')/\delta} + \frac{\operatorname{sh}\pi(x+x')/\delta}{\operatorname{ch}\pi(x+x')/\delta - \cos\pi(y-y')/\delta} + \frac{\operatorname{sh}\pi(x-x')/\delta}{\operatorname{ch}\pi(x-x')/\delta - \cos\pi(y+y')/\delta} + \frac{\operatorname{sh}\pi(x-x')/\delta}{\operatorname{ch}\pi(x-x')/\delta - \cos\pi(y-y')/\delta}\right]$$

As in the case of Sec. 2, the problem reduces to an integral equation for B_y and a definite integral for B_x.

5. Some Generalizations and Numerical Calculations. The procedure developed above can be applied for arbitrary channels with ideally ferromagnetic walls ($\mu_c \to \infty$), allowing mappings onto canonical regions

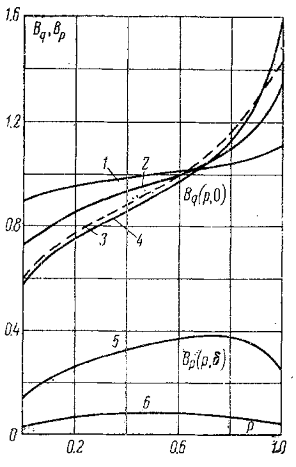

Fig. 1

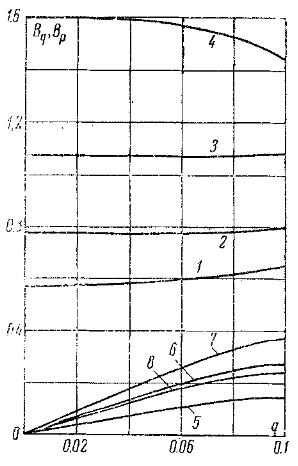

Fig. 2

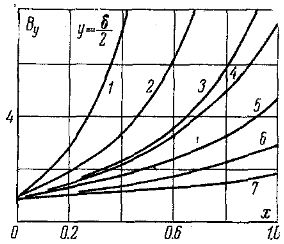

Fig. 3

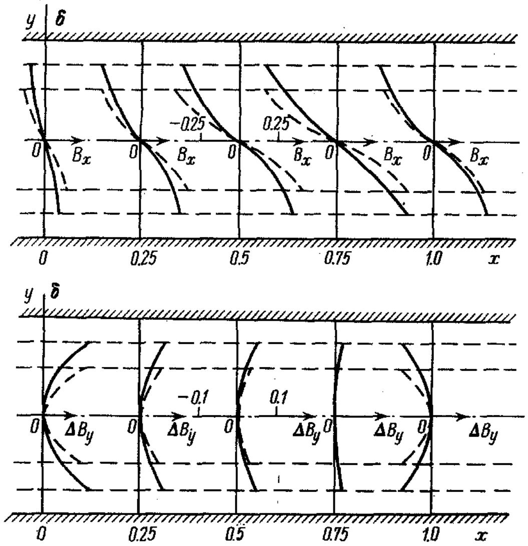

Fig. 4

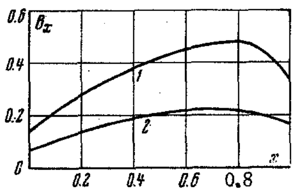

Fig. 5

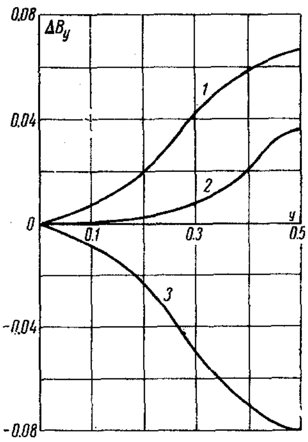

Fig. 6

and for channels with walls having finite permeability if one succeeds in constructing a complex analytic influence function L(P, M) describing the field at a point P due to a unit current at M. Methods for the construction of L for various problems with $\mu_c \to \infty$ and with finite μ_c are well known in applied electricity ([9, 10] et al.). Thus, the question concerns a Fredholm integral representation of a nonanalytic function (fields for $R_m \gg 1$) in terms of analytic functions, the theory of which has been developed quite fully.

In a number of plane problems with $\mu_c \to \infty$ the integral representation for B can be obtained by using the Green's function G for the vector potential, the components of L being expressed in terms of derivatives of G with respect to the longitudinal and transverse coordinates. However, in problems involving steel walls G must be constructed for a Neumann problem (because of the condition requiring the tangential component of B to vanish on the wall) and this often leads to difficulties. For finite μ_c the derivation of G is even more complicated. For example, construction of G for the model of Sec. 1 with commensurate μ_1 and μ_2 is a difficult problem.

In every case the theory developed above, based on physical ideas, gives a more direct and general way of solving problems.

If the analytic determination of the function L is impossible, it can be found approximately in the following way. Into a MHD channel without the working medium (or into a model that is geometrically similar to it) a current-carrying loop is introduced that corresponds in shape to a current element in the channel with its electrodes, and L is found for a selected mesh of points with the loop placed at various positions by direct measurement of the field of the loop.

As illustration of the results obtained, calculations of the fields of the problems of Secs. 1 and 2 were carried out. It was assumed that $B_e = B_0 = \text{const}$, in the steel $\mu \to \infty$, and that the velocity of the fluid had only one component and was parallel to the steel walls. The total field was determined only for the region where $j \neq 0$.

In the problem of Sec. 1 it was assumed that $E = 0$ (short-circuit or axisymmetric model)

$$\delta = \delta_0 + p \operatorname{arc\,tg} x, \quad \operatorname{tg}\alpha \ll 1, \quad \sigma = \text{const}, \quad u_p = 1/\delta(p)$$

Results of calculations on a "NAIRI-2" computer for the case $\delta_0 = 0.1$ are shown in Figs. 1 and 2. Figure 1 has curves for $B_q(p = 0)$ (curve 1 for $R_m = 1$, 2 for $R_m = 3$, 3 and 4 for $R_m = 5$; solid curves for $\alpha = 0$, dashed for $\alpha = 7°$) and curves for $B_p(p,\delta)$ (curve 5 for $R_m = 5$, 6 for $R_m = 1$). With increasing R_m demagnetization at the entrance and magnetization at the exit are increased, and the maximum of $B_q(p, \delta)$ moves toward the exit. Figure 2 shows curves for $B_q(q)$ (curves 1-4) and for $B_p(q)$ (curves 5-8) at $R_m = 5$ and $\alpha = 0$ (curves 1 and 5 for $x = 0$, 2 and 6 for $x = 0.25$, 3 and 7 for $x = 0.75$, 4 and 8 for $x = 1$). The character of the change in $B_q(q)$ at the entrance is opposite to that at the exit, and $B_p(q)$ increases with distance from the steel wall.

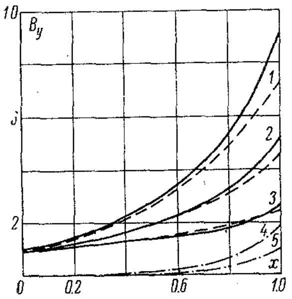

Fig. 7

In the solution of the problem of Sec. 2 a case was considered in which the flow (piston) was separated from the steel walls by nonmagnetic clearances of width Δ. This problem, in addition to instances specified earlier, may be of interest in connection with the motion of a plane free jet between steel pole pieces. It was assumed that

$$\sigma = \text{const}, \ \delta = 1, \ \varepsilon = 0, \ u = u_x \ (y).$$

Results of the calculations are shown in Figs. 3-7. Figure 3 has curves for $B_y(x, \delta/2)$ with $\gamma = 2$ (curve 1 for $R_m = 5$ and $\Delta = 0$, 2 for $R_m = 3$ and $\Delta = 0$, 3 for $R_m = 3$ and $\Delta = 0.125$, 4 for $R_m = 2$ and $\Delta = 0.5$, 5 for $R_m = 3$ and $\Delta = 0.25$, 6 for $R_m = 2$ and $\Delta = 0.25$, 7 for $R_m = 2$ and $\Delta = 0.375$). Growth of $B_y(x)$ is more rapid for larger R_m and smaller Δ. Figure 4 has curves for $B_x(y)$ and $\Delta B_y(y) = [B_y(y) - B_y(\delta/2)]/B_y(\delta/2)$ with $R_m = 2$ at various cross sections of the channel ($x = 0, 0.25, 0.5, 0.75, 1$). The dashed curves correspond to $\Delta = 0.25$ and the solid curves to $\Delta = 0.125$. The scales of B_x and ΔB_y are referred to the cross section at $x = 0.5$. Figure 5 has curves of $B_x(x)$ with $R_m = 2$ for different values of $y = \text{const.}$ (small curve 1 for $y = 0.75\delta$, 2 for $y = 0.625\delta$).

In the case of an inhomogeneous velocity profile $u_x(y)$ the magnetic field in the channel is also inhomogeneous even for $\Delta = 0$. This conclusion is confirmed by curves of $\Delta B_y(y)$ for various values of x and a Poiseuille profile $u_x(y)$, shown in Fig. 6 (curve 1 for $x = 0$, 2 for $x = 0.5$, 3 for $x = 1$; the plane $y = 0$ in Fig. 6 contains the axis of the channel). In this case the character of the change in $\Delta B_y(y)$ at the entrance is opposite to that at the exit. In Fig. 7 the three upper solid curves of $B_y(x, \delta/2)$ were constructed for a Poiseuille profile $u_x(y)$, $R_m = 2$, $\Delta = 0$ and $\gamma = 2$ and for various values of $\varkappa$ (curve 1 for $\varkappa = 0$, 2 for $\varkappa = 0.2$, 3 for $\varkappa = 0.5$). As $\varkappa$ increases, the role of the induced field diminishes. For comparison similar (dashed) curves of $B_y(x, \delta/2)$ for a homogeneous profile giving the same mean flow rate are shown in the same figure. It is evident that the field distortion is larger for the Poiseuille profile. The dashed curves were calculated from an integral equation and one-dimensional theory (see, for instance, [5]). For $R_m \le 5$ the disagreement did not exceed 3%. Curve 4 in Fig. 7 was constructed for $\Delta = 0$, $u_x = \text{const}$, $\sigma = \text{const}$, $\varepsilon = 0$, $R_m = 5$, $\gamma = 0$ (symmetric external circuit), and curve 5, for the same conditions except with $\gamma = -2$ (external circuit to the left).

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